Minimal Spanning Tree
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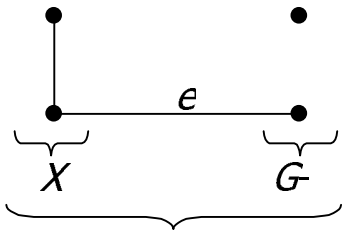
A Minimal Spanning Tree (MST) is a subgraph this is connected and whose sum of its edges is minimized.

Lemma: If X is a subset of vertices in G, and e is the smallest edge connecting X to G–X, then e is part of the MST.



Kruskal's Algorithm:

```
sort the edges of G in increasing order by length
keep a subgraph S of G, initially empty
for each edge e in sorted order
  if the endpoints of e are disconnected in S
    add e to S
return S
```

Prim's Algorithm:

```
let T be a single vertex x
  while (T has fewer than n vertices)
    find the smallest edge connecting T to G-T
    add it to T
```

Prim can be speeded up by using a heap to remember, for each vertex, the smallest edge connecting T with that vertex.

Kruskal	Prim	Prim (heap)
O (m log m)	O (n^2)	O (m log n)